BELAJAR BERSAMA PAK RIDWAN UNTUK SISWA SD DAN SMP SECARA ONLINE

HOME


HOME

HOME

Sesi 2: Bilangan - Teleskopik

Terima Kasih

Meningkatkan Kecerdasan Otak di Dada

Teleskopik Penjumlahan

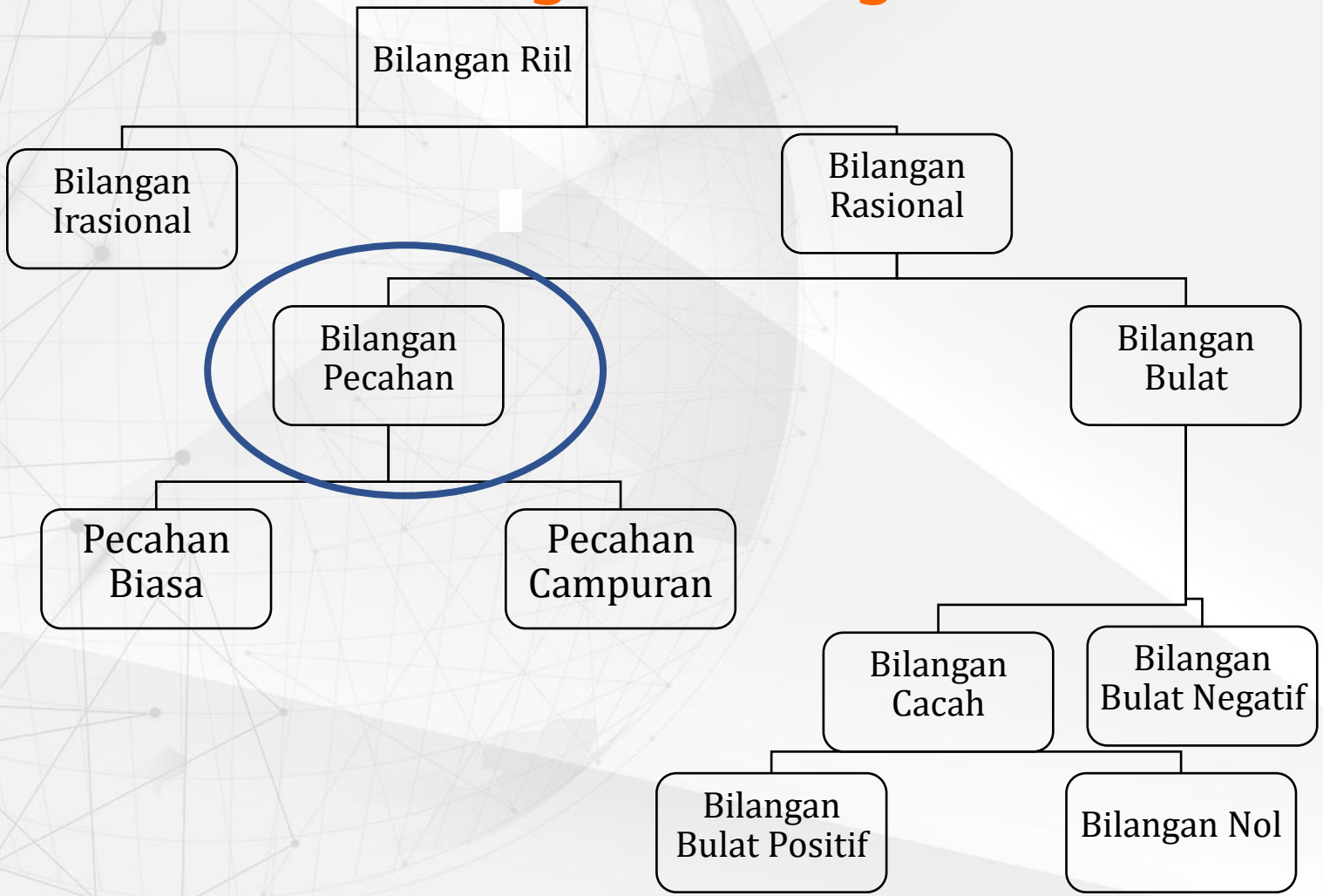
TEGAS PUAS KPM IKHLAS

READI HUMAN SCHOOL

Meningkatkan Kecerdasan Otak di Dada

Jenis Bilangan

Diagram Bilangan



HOME

Teleskopik Penjumlahan

Deret teleskopik penjumlahan adalah deret yang suku-sukunya dapat dipecah ke bentuk tertentu, agar penjumlahan suku-sukunya dapat saling menghilangkan sehingga hanya menyisakan suku pertama dan terakhir.

Misal terdapat penjumlahan deret $a_1 + a_2 + a_3 + \cdots + a_n = \sum_{k=1}^{n} a_k$.

Jika penjumlahan deret di atas diubah ke dalam bentuk lain dengan dasar kesamaan $a_k = V_k - V_{k-1}$, maka akan terbentuk

$$\sum_{k=1}^{n} a_k = (V_1 - V_0) + (V_2 - V_1) + \cdots + (V_n - V_{n-1}) = V_n - V_0$$

Teleskopik Penjumlahan

Jika

$$\frac{b-a}{a \times b} = \frac{1}{a} - \frac{1}{b}$$

Maka untuk nilai:

$$\frac{1}{a \times b} = \frac{1}{b-a} \cdot \left(\frac{1}{a} - \frac{1}{b}\right)$$

Contoh:

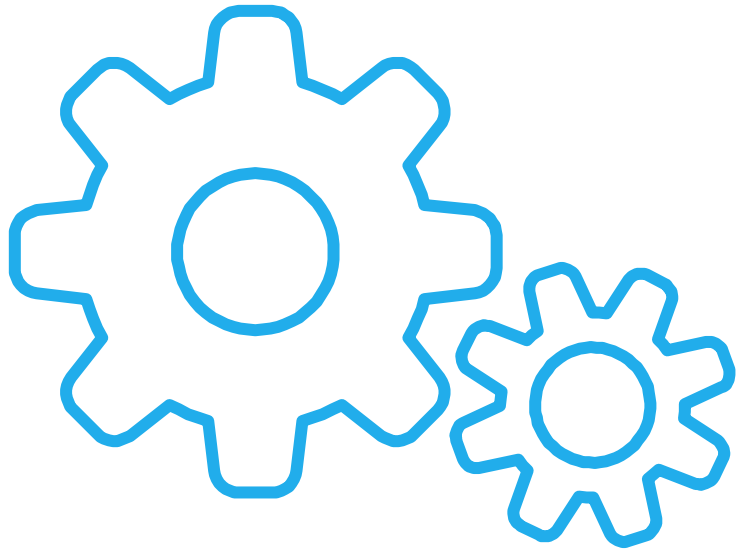
$$\frac{1}{4 \times 5} = \frac{1}{5-4} \times \left(\frac{1}{4} - \frac{1}{5}\right)$$

$$= \frac{1}{1} \times \left(\frac{1}{4} - \frac{1}{5}\right) = \frac{1}{4} - \frac{1}{5}$$

$$\frac{1}{3 \times 5} = \frac{1}{5-3} \times \left(\frac{1}{3} - \frac{1}{5}\right)$$

$$= \frac{1}{2} \times \left(\frac{1}{3} - \frac{1}{5}\right)$$

HOME



Soal Latihan

Teleskopik Penjumlahan

Latihan Soal 1

KTA

Teleskopik Penjumlahan

$$\frac{1}{a\times b}=\frac{1}{b-a}.\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots+\frac{1}{8\times9}=\ldots.$$

Solusi

$$\frac{1}{1\times2}=\frac{1}{2-1}\left(\frac{1}{1}-\frac{1}{2}\right)=\frac{1}{1}\left(\frac{1}{1}-\frac{1}{2}\right)=\frac{1}{1}-\frac{1}{2}$$

$$\frac{1}{2\times3}=\frac{1}{3-2}\left(\frac{1}{2}-\frac{1}{3}\right)=\frac{1}{1}\left(\frac{1}{2}-\frac{1}{3}\right)=\frac{1}{2}-\frac{1}{3}$$

$$\frac{1}{3\times4}=\frac{1}{3}-\frac{1}{4}$$ dan seterusnya sampai

$$\frac{1}{8\times9}=\frac{1}{8}-\frac{1}{9}$$

Maka dapat kita buat

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots+\frac{1}{8\times9}=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{8}-\frac{1}{9}\right)$$

$$=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{8}-\frac{1}{9}\right)$$

$$=\frac{1}{1}-\frac{1}{9}$$

$$=\frac{8}{9}$$

HOME

Latihan Soal 2

KTA

Teleskopik Penjumlahan

$$\frac{1}{a\times b}=\frac{1}{b-a}.\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$\frac{1}{3\times5}+\frac{1}{5\times7}+\frac{1}{7\times9}+\ldots+\frac{1}{97\times99}=\ldots$$

Solusi

$$\frac{1}{3\times5}=\frac{1}{5-3}\left(\frac{1}{3}-\frac{1}{5}\right)=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)$$

$$\frac{1}{5\times7}=\frac{1}{7-5}\left(\frac{1}{5}-\frac{1}{7}\right)=\frac{1}{2}\left(\frac{1}{5}-\frac{1}{7}\right)$$

dan seterusnya sampai

$$\frac{1}{97\times99}=\frac{1}{99-97}\left(\frac{1}{97}-\frac{1}{99}\right)=\frac{1}{2}\left(\frac{1}{97}-\frac{1}{99}\right)$$

Maka dapat kita buat

$$\frac{1}{3\times5}+\frac{1}{5\times7}+\frac{1}{7\times9}+\ldots+\frac{1}{97\times99}=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)+\frac{1}{2}\left(\frac{1}{5}-\frac{1}{7}\right)+\ldots+\frac{1}{2}\left(\frac{1}{97}-\frac{1}{99}\right)$$

$$=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\ldots+\frac{1}{97}-\frac{1}{99}\right)$$

$$=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{99}\right)$$

$$=\frac{16}{99}$$

HOME

Latihan Soal 3

KTA

Teleskopik

$$\frac{1}{a\times b}=\frac{1}{b-a}.\left(\frac{1}{a}-\frac{1}{b}\right)$$

Deret Aritmatika

$$1+2+3+\ldots+n=\frac{n(n+1)}{2}$$

Hitunglah hasil dari: $\frac{3}{2}+\frac{13}{6}+\frac{37}{12}+\frac{81}{20}+\ldots+\frac{980101}{9900}$

Solusi

Perhatikan bahwa $\frac{3}{2}=1\frac{1}{2}=1+\frac{1}{2}$. Diperoleh:

$$\frac{3}{2}+\frac{13}{6}+\frac{37}{12}+\frac{81}{20}+\ldots+\frac{980101}{9900}$$

$$=1+\frac{1}{2}+2+\frac{1}{6}+3+\frac{1}{12}+4+\frac{1}{20}+\ldots+99+\frac{1}{9900}$$

$$=(1+2+3+\ldots+99)+\left(\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\ldots+\frac{1}{9900}\right)$$

$$=4950+\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{99}-\frac{1}{100}\right)$$

$$=4950+\left(\frac{1}{1}-\frac{1}{100}\right)$$

$$=4950+\frac{99}{100}$$

$$=4950\frac{99}{100}$$

HOME

Latihan Soal 4

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Teleskopik Penjumlahan

$$\frac{1}{a\times b}=\frac{1}{b-a}.\left(\frac{1}{a}-\frac{1}{b}\right)$$

Diberikan $x=101!\times\left(\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{99}{100!}\right)$. Tentukan nilai dari $101!-x$.

Solusi

Misalkan $R=\left(\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots+\frac{99}{100!}\right)$

Perhatikan bahwa $\frac{n}{(n+1)!}=\frac{(n+1)-1}{(n+1)!}=\frac{1}{n!}-\frac{1}{(n+1)!}$, maka

$$R=\left(\frac{1}{1!}-\frac{1}{2!}\right)+\left(\frac{1}{2!}-\frac{1}{3!}\right)+\left(\frac{1}{3!}-\frac{1}{4!}\right)+\ldots+\left(\frac{1}{99!}-\frac{1}{100!}\right)=1-\frac{1}{100!}$$

Dengan demikian,

$$101!-x=101!-101!\times\left(1-\frac{1}{100!}\right)$$

$$=\frac{101!}{100!}$$

$$=101$$

HOME

KTA

Teleskopik

$$\frac{a+b}{a\times b}=\frac{1}{a}+\frac{1}{b}$$

Latihan Soal 5

$$\frac{5}{2\times3}-\frac{7}{3\times4}+\frac{9}{4\times5}-\frac{11}{5\times6}+\ldots-\frac{199}{99\times100}=\ldots.$$

Solusi

$$\frac{5}{2\times3}-\frac{7}{3\times4}+\frac{9}{4\times5}-\frac{11}{5\times6}+\ldots-\frac{199}{99\times100}$$

$$=\frac{1}{2}+\frac{1}{3}-\left(\frac{1}{3}+\frac{1}{4}\right)+\frac{1}{4}+\frac{1}{5}-\left(\frac{1}{5}+\frac{1}{6}\right)+\ldots-\left(\frac{1}{99}+\frac{1}{100}\right)$$

$$=\frac{1}{2}-\frac{1}{100}=\frac{50}{100}-\frac{1}{100}=\frac{49}{100}$$

HOME

Latihan Soal 6

$$\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\ldots+\frac{1}{100\times101\times102}=\ldots.$$

Solusi

$$\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\ldots+\frac{1}{100\times101\times102}$$

$$=\frac{1}{2}\left(\frac{1}{1\times2}-\frac{1}{2\times3}\right)+\frac{1}{2}\left(\frac{1}{2\times3}-\frac{1}{3\times4}\right)+\ldots+\frac{1}{2}\left(\frac{1}{100\times101}-\frac{1}{101\times102}\right)$$

$$=\frac{1}{2}\left[\left(\frac{1}{1\times2}-\frac{1}{2\times3}\right)+\left(\frac{1}{2\times3}-\frac{1}{3\times4}\right)+\ldots+\left(\frac{1}{100\times101}-\frac{1}{101\times102}\right)\right]$$

KTA

Teleskopik

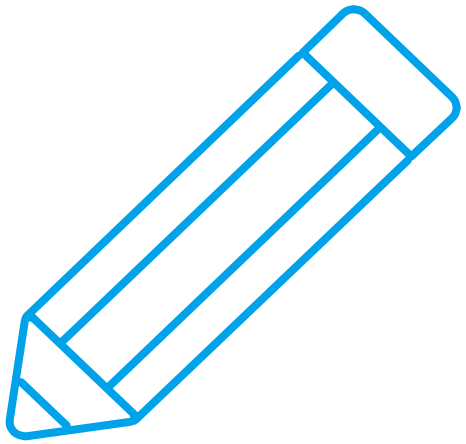
$$\frac{1}{a\times b\times c}=\frac{1}{2}\left(\frac{1}{a\times b}-\frac{1}{b\times c}\right)$$

dimana *a*, *b*, dan *c* adalah bilangan berurutan

$$=\frac{1}{2}\left[\frac{1}{2}-\frac{1}{101\times102}\right]$$

$$=\frac{2575}{10302}$$

HOME



Teleskopik Perkalian

Teleskopik Perkalian

Deret teleskopik perkalian adalah deret yang setiap sukunya dapat diubah ke bentuk tertentu, agar perkalian suku-sukunya saling menghilangkan pembilang dan penyebut sehingga menyisakan pembilang suku awal dan penyebut suku terakhir atau sebaliknya.

Misal terdapat perkalian deret:

$$a_1 \times a_2 \times a_3 \times \cdots \times a_n$$

Jika perkalian deret di atas diubah ke dalam bentuk sederhana dengan dasar kesamaan $a_k = \frac{t_k}{t_{k+1}}$, maka akan terbentuk

$$a_1 \times a_2 \times a_3 \times \cdots \times a_n = \frac{t_1}{t_2} \times \frac{t_2}{t_3} \times \frac{t_3}{t_4} \times \cdots \times \frac{t_n}{t_{n+1}} = \frac{t_1}{t_{n+1}}$$

Soal Latihan

Teleskopik Perkalian

Latihan Soal 7

KTA

Teleskopik Perkalian

$$\frac{t_1}{t_2}\times\frac{t_2}{t_3}\times\frac{t_3}{t_4}\times\ldots\times\frac{t_n}{t_{n+1}}=\frac{t_1}{t_{n+1}}$$

Hasil dari $\sqrt{1+\frac{1}{3}}\cdot\sqrt{1+\frac{1}{4}}\cdot\sqrt{1+\frac{1}{5}}\cdot\ldots\cdot\sqrt{1+\frac{1}{1874}}$ adalah ....

Solusi

Jumlahkan setiap bilangan dalam tanda akar dan gunakan sifat akar bahwa $\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$, diperoleh

$$\sqrt{\frac{4}{3}}\cdot\sqrt{\frac{5}{4}}\cdot\sqrt{\frac{6}{5}}\cdot\ldots\cdot\sqrt{\frac{1875}{1874}}$$

$$=\sqrt{\frac{4}{3}\cdot\frac{5}{4}\cdot\frac{6}{5}\cdot\ldots\cdot\frac{1875}{1874}}$$

$$=\sqrt{\frac{\cancel{4}}{3}\cdot\frac{\cancel{5}}{\cancel{4}}\cdot\frac{\cancel{6}}{\cancel{5}}\cdot\ldots\cdot\frac{1875}{\cancel{1874}}}$$

$$=\sqrt{\frac{1875}{3}}$$

$$=\sqrt{625}=25$$

Latihan Soal 8

KTA

Teleskopik Perkalian

$$\frac{t_1}{t_2}\times\frac{t_2}{t_3}\times\frac{t_3}{t_4}\times\ldots\times\frac{t_n}{t_{n+1}}=\frac{t_1}{t_{n+1}}$$

Diketahui $\sqrt{\frac{5}{a}\times\frac{6}{5}\times\frac{7}{6}\times\frac{8}{7}\times\ldots\times\frac{b}{c}}=2$. Nilai dari $b+ac=\ldots.$

Solusi

Perhatikan pola deret pada soal. Nilai pembilang satu lebihnya dari penyebut, maka diperoleh nilai a adalah 4 dan $b = c + 1$.

Dari persamaan yang diberikan, diperoleh

$$\sqrt{\frac{5}{a}\times\frac{6}{5}\times\frac{7}{6}\times\frac{8}{7}\times\ldots\times\frac{b}{c}}=2.$$

$$\sqrt{\frac{\cancel{5}}{a}\times\frac{\cancel{6}}{\cancel{5}}\times\frac{\cancel{7}}{\cancel{6}}\times\frac{\cancel{8}}{\cancel{7}}\times\ldots\times\frac{b}{\cancel{c}}}=2.$$

$$\sqrt{\frac{b}{a}}=2$$

$$\frac{b}{a}=4$$

$$b=4a$$

Karena $a = 4$, maka $b = 16$ dan akibatnya $c = 15$.

Jadi, nilai dari

$$b + ac = 16 + 4 \cdot 15 = 76$$

Soal Tantangan

KTA

Teleskopik Penjumlahan

$$\frac{1}{a\times b}=\frac{1}{b-a}.\left(\frac{1}{a}-\frac{1}{b}\right)$$

Persamaan Kuadrat

$$a^2-b^2=(a+b)(a-b)$$

Soal Tantangan 1

Jika $f(x)=4x^2-1$, tentukan hasil dari $\frac{1}{f(1)}+\frac{1}{f(2)}+\frac{1}{f(3)}+\ldots+\frac{1}{f(84)}$.

Solusi

Perhatikan bahwa $f(x)=4x^2-1=(2x+1)(2x-1)$, sehingga

$$\frac{1}{f(x)}=\frac{1}{4x^2-1}=\frac{1}{(2x+1)(2x-1)}=\frac{1}{2}\left(\frac{1}{2x-1}-\frac{1}{2x+1}\right)$$

Dengan demikian, kita peroleh

$$\frac{1}{f(1)}+\frac{1}{f(2)}+\frac{1}{f(3)}+\ldots+\frac{1}{f(84)}.$$

$$=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{3}\right)+\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)+\frac{1}{2}\left(\frac{1}{5}-\frac{1}{7}\right)+\ldots+\frac{1}{2}\left(\frac{1}{167}-\frac{1}{169}\right)$$

$$=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\ldots+\frac{1}{167}-\frac{1}{169}\right)$$

$$=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{169}\right)$$

$$=\frac{1}{2}\left(\frac{168}{169}\right)=\frac{84}{169}$$

HOME

Soal Tantangan 2

KTA

Sifat Distributif

$$a\times(b+c)=(a\times b)+(a\times c)$$

Berapa bentuk sederhana dari

$$\frac{1}{1\times2017}+\frac{1}{2\times2016}+\ldots+\frac{1}{2017\times1}-\frac{2017}{2018}\left(\frac{1}{1\times2016}+\frac{1}{2\times2015}+\ldots+\frac{1}{2016\times1}\right)$$

Solusi

Diketahui bahwa jumlah penyebut pada pola tersebut adalah sama, maka kita peroleh

$$\frac{1}{1\times2017}+\frac{1}{2\times2016}+\ldots+\frac{1}{2017\times1}-\frac{2017}{2018}\left(\frac{1}{1\times2016}+\frac{1}{2\times2015}+\ldots+\frac{1}{2016\times1}\right)$$

$$=\frac{1}{2018}\left[\frac{2018}{1\times2017}+\frac{2018}{2\times2016}+\ldots+\frac{2018}{2017\times1}-\left(\frac{2017}{1\times2016}+\frac{2017}{2\times2015}+\ldots+\frac{2017}{2016\times1}\right)\right]$$

$$=\frac{1}{2018}\left[\frac{1}{1}+\frac{1}{2017}+\frac{1}{2}+\frac{1}{2016}+\mathrm{L}\ +\frac{1}{2017}+\frac{1}{1}-\left(\frac{1}{1}+\frac{1}{2016}+\frac{1}{2}+\frac{1}{2015}+\mathrm{L}\ +\frac{1}{2016}+\frac{1}{1}\right)\right]$$

$$=\frac{1}{2018}\left(\frac{1}{2017}+\frac{1}{2017}\right).$$

$$=\frac{1}{1009\times2017}=\frac{1}{2035153}$$

Soal Tantangan 3

Hitunglah hasil dari

$$\frac{1}{1\cdot 6}+\frac{1}{2\cdot 9}+\frac{1}{3\cdot 12}+\ldots+\frac{1}{195\cdot 588}$$

Solusi

$$\frac{1}{1\cdot 6}+\frac{1}{2\cdot 9}+\frac{1}{3\cdot 12}+\ldots+\frac{1}{195\cdot 588}=\frac{1}{3}\left(\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{195\cdot 196}\right)$$

$$=\frac{1}{3}\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{195}-\frac{1}{196}\right)$$

$$=\frac{1}{3}\left(\frac{195}{196}\right)=\frac{65}{196}$$

HOME

Soal Tantangan 4

Tentukan bentuk sederhana dari

$$\frac{100!+99!}{100!-99!}\times\frac{98!+97!}{98!-97!}\times\frac{96!+95!}{96!-95!}\times\ldots\times\frac{2!+1!}{2!-1!}$$

Solusi

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